Dynamics of four- versus two-terminal transport through chaotic quantum cavities

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We consider multi-terminal mesoscopic transport through a well-conducting chaotic quantum cavity using random matrix theory. Four-probe resistance vanishes on the average and is not affected by weak localization. Its fluctuations are given by a single expression valid for arbitrary temperature, ac frequency, non-ideal coupling of the contacts, and in the presence of floating probes; surprisingly, they are governed by the dwell time only. In contrast, the two-probe transport additionally depends on the RC-time, which is interpreted as a property of the measurement scheme. We also predict a universal mesoscopic distribution of the phase of transmitted voltage in an ac experiment.

Introduction. Mesoscopic transport and its sample-to-sample fluctuations are fundamentally important for understanding quantum effects in electronic transfer through nano-structures. However, when a voltage is measured between the source and drain in a two-terminal geometry, a big classical contribution to transport shadows quantum effects of order e^2/h [1, 2]. The visibility of interference effects can be enhanced by diminishing the width of the contacts to reservoirs, but this also leads to Coulomb blockade of electrons [2]. Another approach is to keep the contacts well-conducting and apply a multi-terminal scheme with a voltage through the sample measured by two *additional* contacts, see Fig. 1(a). The thus defined four-terminal resistance R_{4t}, well-known to experimentalists, has some benefits over the two-terminal one, R_{2t}, which is more popular among theoreticians.

The central advantage of R_{4t} is to exclude the contact properties. Indeed, classical resistances are local, and a four-terminal scheme removes the effect of the contacts and therefore probes the sample's properties only [3]. Quantum mechanically resistance is not local, and this scheme does reduce the role of contacts but they still affect the measurement. Nevertheless even for a coherent sample there are of course profound differences between R_{2t} and R_{4t}. The former determines Joule heating and is therefore always positive and symmetric to the magnetic field inversion. In contrast, the latter need not be positive [4, 5] and experiments demonstrate its fluctuations around zero [6–8] and magnetic field asymmetry [9].

Non-universal fluctuations in R_{4t} were widely investigated in metallic diffusive samples, where small quantum effects on top of a large classical average depend on the probe locations [10–15]. On the other hand, in a chaotic cavity [16, 17] the exact positions of probes are irrelevant, the classic voltage drop vanishes, and an experiment would measure quantum effects directly. Remarkably, R_{4t} in this generic geometry remained unexplored except for a recent initial experiment [18].

What could one expect from R_{4t} in a quantum dot? Its

dc two-terminal conductance shows a weak-localization correction (WL) $\sim e^2/h$ due to an enhanced return probability for an electron, and universal conductance fluctuations (UCF) $\sim (e^2/h)^2$; a magnetic field destroys WL and reduces UCF by a factor 2 [16]. These results for ballistic quantum point contacts (QPCs) can be generalized to include *either* (i) a dephasing (floating) probe coupling [19], or (ii) non-ideal QPCs [20], or (iii) a low-frequency ac setup [21]. In (iii) the averaged conductance [21] and shot-noise [22] in their leading order depend only on the charge relaxation time (RC-time) τ_{RC}, sensitive to the Coulomb interaction energy $e^2/2C$ of the capacitor. Its appearance is natural: at high frequencies $\omega \gtrsim 1/\tau_{RC}$ the capacitor conducts better than the contacts. However, WL and UCF, as well as the third current cumulant, additionally depend on the dwell time τ_d an electron typically spends in the dot [21, 23, 24]. As a result, even in linear transport there is no unique time-scale for dispersion of various quantities.

At first one expects R_{4t} to share the features of two-terminal WL and UCF. However, these expectations are
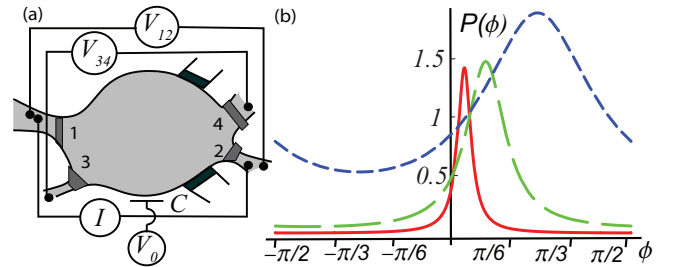

FIG. 1: (a) A multi-terminal chaotic cavity with a gate (capacitance C). A current through source-drain contacts 1,2 and a voltage drop between 3,4 (or 1,2) determine the four-probe $R_{4t} = V_{34}/I$ (or two-probe $R_{2t} = V_{12}/I$) resistances. Additional floating probes draw no current. (b) The mesoscopic distribution $P(\phi)$ (rescaled by $\times \omega\tau_d$) of phase ϕ of the transmitted ac voltage as a function of frequency, for $\omega\tau_d = 0.1, 0.3, 1$, full, long- and short-dashed, respectively.

wrong: $R_{4\mathrm{t}}$ does not have weak localization, and its fluctuations are insensitive to magnetic field. The differences are even greater when the dynamics of resistance fluctuations is compared: unlike $R_{2\mathrm{t}}$ governed both by τ_{d} and τ_{RC}, $R_{4\mathrm{t}}$ turns out to depend only on τ_{d}. Surprisingly, fluctuations of $R_{4\mathrm{t}}$ are much simpler than UCF, and a single formula covers the regimes (i-iii) for arbitrary floating probes, imperfections in contacts, and frequency ω.

Universal resistance fluctuations. This Letter compares statistics of two- and four-terminal resistances in quantum dots with non-ideal coupling to reservoirs and probes, biased by ac voltages, see Fig. 1(a). To be specific let the current I flow only through contacts 1,2 and measure the voltage drop between 3,4 (or 1,2) to find $R_{4\mathrm{t}} = V_{34}/I$ (or $R_{2\mathrm{t}} = V_{12}/I$). Each contact j with N_j orbital channels is specified by a diagonal matrix Γ_j of its channel transmissions, and we assume good dimensionless conductances of the probes 1–4, $\mathrm{tr}\,\Gamma_j \gg 1$. At arbitrary temperature T and measurement frequency ω, in first two orders in the total QPC conductance $\mathrm{tr}\,\Gamma$, the ensemble averaged resistances are given by

$$\langle R_{4\mathrm{t}} \rangle = 0, \tag{1}$$

$$\langle R_{2\mathrm{t}} \rangle = \frac{h}{2e^2} \frac{\mathrm{tr}\,\Gamma_1 + \mathrm{tr}\,\Gamma_2}{\mathrm{tr}\,\Gamma_1 \mathrm{tr}\,\Gamma_2} \left[1 + \frac{\delta_{\beta 1}}{\mathrm{tr}\,\Gamma(1 - i\omega\tau_{\mathrm{d}})} \right]. \tag{2}$$

Vanishing $\langle R_{4\mathrm{t}} \rangle$ is not sensitive to the presence of time-reversal symmetry (TRS), $\beta = 1$, or its breaking by magnetic field, $\beta = 2$. In contrast, $\langle R_{2\mathrm{t}} \rangle$ is corrected by WL depending only on τ_{d}. Similarly, TRS does (not) affect fluctuations of $R_{2\mathrm{t}}(R_{4\mathrm{t}})$ around their averages:

$$\mathrm{Var}\,R_{4\mathrm{t}} = \left(\frac{h}{2e^2 \mathrm{tr}\,\Gamma} \right)^2 \sum_{\substack{i=1,2 \\ v=3,4}} \frac{\mathrm{tr}\,\Gamma_i^2}{\mathrm{tr}^2\Gamma_i} \frac{\mathrm{tr}\,\Gamma_v^2}{\mathrm{tr}^2\Gamma_v} \int_{\omega\omega}, \tag{3}$$

$$\mathrm{Var}\,R_{2\mathrm{t}} = \frac{2}{\beta} \left(\frac{h}{2e^2} \frac{N_1 + N_2}{N_1 N_2 N} \right)^2 \frac{1 - 2i\omega\tau_{\mathrm{RC}}}{(1 - i\omega\tau_{\mathrm{RC}})^2} \int_{\omega\omega}, \tag{4}$$

$$\int_{\omega\Omega} \equiv \int_0^{+\infty} \left(\frac{2\pi T \sin \omega\tau/2}{\hbar\omega \sinh \pi T\tau/\hbar} \right)^2 \frac{e^{\tau(i\Omega - 1/\tau_{\mathrm{d}})} d\tau}{(1 - i\Omega\tau_{\mathrm{d}})\tau_{\mathrm{d}}} \tag{5}$$

Fluctuations of $R_{4\mathrm{t}}$, Eq. (3), the main result of this paper, are valid for any couplings of floating probes, and imperfections in the contacts, while Eq. (4) is derived only for ballistic QPCs in a multi-terminal dot.

In an effectively zero-dimensional dot universal Eqs. (1–4) do not depend on disorder and contact positions. It is interesting that for conductance there are no results of comparable generality for WL and UCF . We conclude that both weak-localization corrections to the two-terminal resistance, Eq. (2), and fluctuations of the four-terminal one, Eq. (3), are *completely insensitive at all frequencies* to the RC-time and thus to Coulomb interactions. We next explain how these results are obtained, and discuss the dc fluctuations of resistances. Later we generalize our results onto an ac setup and discuss the role of the chosen measurement scheme in fluctuations.

We calculate the universal distribution of the phase of transmitted voltage in a four-terminal ac experiment.

The calculation. We consider a chaotic quantum dot in a multi-probe setup with $M \geq 4$ contacts and a gate with capacitance C, see Fig. 1 (a). Each contact $j = 1, ..., M$ leading to a reservoir with a voltage $V_{j\omega}$ at the frequency ω is characterized by a set of transmission values Γ_j of its (imperfect) channels. This diffusive/ballistic dot is in the universal regime, when direct trajectories are absent, and chaos validates random matrix theory (RMT) [16, 17]. The Thouless energy of a closed dot is much larger than the temperature T, mean level spacing $\Delta = 2\pi\hbar^2/(m\,\mathrm{Area})$, the excitation energy $\hbar\omega$ in the ac experiment, and escape rate $\mathrm{tr}\,\Gamma \cdot \Delta/2\pi$. The total coupling to environment is good, $\mathrm{tr}\,\Gamma \gg 1$, and it specifies the electronic dwell-time $\tau_{\mathrm{d}} = h/(\mathrm{tr}\,\Gamma\Delta)$ in the dot. The leading Coulomb interaction effect is accounted for by a uniform self-consistent potential of the dot, found from charge conservation [21, 25]. The terms leading to Coulomb blockade are unimportant due to $1/\mathrm{tr}\,\Gamma \ll 1$. We use RMT and diagrammatic technique with a small parameter $1/\mathrm{tr}\,\Gamma \ll 1$ for an energy-dependent scattering matrix $\mathcal{S}(\varepsilon)$ of a dot with imperfect contacts [20, 26, 27] to find transport statistics. Further we take $h = e = 1$ and consider spin-degenerate electrons, $\nu_s = 2$, the role of spin-orbit in $R_{4\mathrm{t}}$ being discussed elsewhere [28].

Linear transport coefficients are found from the screened frequency-dependent degenerate $(M+1)\times(M+1)$ conductance matrix $g_{ij,\omega}^s = \partial I_{i\omega}/\partial V_{j\omega}$, where

$$g_{ij,\omega}^s = g_{ij,\omega} - \sum_{kl} g_{il,\omega} g_{kj,\omega}/(\sum_{kl} g_{kl,\omega} - i\omega C),$$

$$g_{ij,\omega} = 2\pi\nu_s \int d\varepsilon \mathrm{tr}\,[\mathbb{1}_i \mathbb{1}_j - \mathbb{1}_j \mathcal{S}^\dagger(\varepsilon)\mathbb{1}_i \mathcal{S}(\varepsilon + \omega/2\pi)]$$
$$\times (f(\varepsilon) - f(\varepsilon + \omega/2\pi))/\omega, \ i,j,k,l = \overline{1,M}, \tag{6}$$

using summation over k or l, $\sum_{k(l)=0}^M g_{kl,\omega}^s = 0$ [21, 25]. All probes except the current source 1 and sink 2, $I = I_1 = -I_2$, are set to voltages such that they draw no current. We shift all voltages by $-V_{4\omega}$ and eliminate the 4th row and column from g^s, invert the rest and obtain the resistance matrix R. It is used to explore statistics of $R_{4\mathrm{t}} = R_{31} - R_{32}$ and $R_{2\mathrm{t}} = R_{11} - R_{12} - R_{21} + R_{22}$.

DC setup, $\omega = 0$. First, let us consider dc transport and ballistic QPCs at $T = 0$, when any $\mathrm{tr}\,\Gamma_j^n = N_j$ in Eqs. (2,3) and the integral, Eq. (5), is reduced to 1. In this limit the measurements in a two-terminal dot are not affected by the Coulomb interaction, and both $\langle R_{2\mathrm{t}} \rangle$ and its fluctuations are diminished by the TRS-breaking [29]. In a multi-terminal setup, however, chaotic scattering results in a random voltage drop between any pair of voltage probes. As a consequence, the average $R_{4\mathrm{t}}$ vanishes, $\langle R_{4\mathrm{t}} \rangle = 0$, and does not have a WL contribution. Indeed, $R_{2\mathrm{t}} = (g_{33}g_{44} - g_{34}g_{43}) \cdot \det R$ is symmetric with respect to $1 \leftrightarrow 2$, and its WL is the first correction due to self-intersections of an electronic trajectory from 1 to 2. On the other hand, $R_{4\mathrm{t}} = (g_{31}g_{42} - g_{32}g_{41}) \cdot \det R$ [4]

is anti-symmetric, and any loop contributes equally to both terms. Consequently, WL corrections are canceled, and they appear only if $\langle R_{4t} \rangle \neq 0$, *e.g.* in a quasi-1d disordered structure [30] with a special probe arrangement.

Similarly, the TRS-breaking does not affect the fluctuations of R_{4t} as shown by Eq. (3), in contrast with Eq. (4) for R_{2t}. Indeed, inverting time we must invert magnetic flux Φ and swap the current- with voltage-probes. The ensuing symmetry $R_{2t}(\Phi) = R_{2t}(-\Phi)$ gives $\partial R_{2t}/\partial \Phi = 0$ at $\Phi = 0$, but there is no such a restriction on $R_{4t}(\Phi)$. Fluctuations of $R_{2t}(\Phi)$ are analogous to a perturbation on a string with a free end, where the perturbation maximum corresponds to the maximum in R_{2t}-fluctuations at $\Phi = 0$. On the other hand, $R_{4t}(\Phi)$ is analogous to an *infinite* string, so that zero field is as good as any other: neither TRS-breaking effects like WL in Eq. (2) or $2/\beta$-coefficient of Eq. (4), nor "quenching" (diminished sensitivity to Φ at $\Phi = 0$) appear for R_{4t} in chaotic dots [28]. In addition, in contrast to the classical four-terminal scheme [3], the properties of source and drain are never fully eliminated: a measurement affects the sample and so the mesoscopic fluctuations in R_{4t} do depend on contacts. Importantly, in the experiment [18] the voltage probes are invasive.

For the non-ideal contact coupling a tunneling limit for the voltage probes is often taken, $\mathrm{tr}\,\Gamma_v \ll 1$; in the opposite limit, $\mathrm{tr}\,\Gamma_v \gg 1$, UCF presents a complicated 6-th order polynomial of Γ_j [20]. In contrast, for a fixed set $\{\Gamma_j\}$ the fluctuations of R_{4t} given by Eq. (3) can be expressed only in terms of conductance $\mathrm{tr}\,\Gamma_j$ and Fano factor $\mathcal{F}_j$, the ratio of shot noise to current [31], of each QPC. The result of a separate averaging over mesoscopic contact j, $\mathrm{tr}\,\Gamma_j^2/\mathrm{tr}\,^2\Gamma_j \to (1 - \langle \mathcal{F}_j \rangle)/\langle \mathrm{tr}\,\Gamma_j \rangle$, is proportional to the QPC resistance with a numerical prefactor: it ranges from 1 for ballistic to $2/3$ for a short diffusive wire, if disorder is increased; for a wide tunneling QPC it is equal to 1 if $\Gamma_j \ll 1$ are the same, and $1/2$ for a dirty interface [32]. We observe that, generally, a poorer coupling increases fluctuations of R_{4t}.

Often conductors have more contacts then needed for a four-terminal measurement. A floating (unused) contact f does not to draw any current from the sample, but leads to decoherence [33], its coupling is related to the inelastic scattering time $\tau_{in} = h/\Delta\mathrm{tr}\,\Gamma_f$ [19]. Data are often interpreted as an addition of several fictitious dephasing channels to the real contacts. However, here it is important to distinguish between a floating probe and a contact used for transport measurements. Similarly to Ref. 19, we find that a floating probe decreases fluctuations of R_{4t} by increasing the total coupling $\mathrm{tr}\,\Gamma$ of the dot to reservoirs. Due to the zero-dimensional geometry of the dot, they are independent of the number of such probes, and only the total $\mathrm{tr}\,\Gamma_f$ is relevant. The voltage probes used to define R_{4t} and floating probes are not equivalent, an increase of the former decreases fluctuations stronger. Indeed, in two measurements with unequal probes used as voltage and floating ones and vice versa, the better the voltage probe conducts the smaller the fluctuations of R_{4t}. Importantly, if the dephasing probe model is used, dephasing channels should be included not into the real, but into the floating probes.

AC setup, $\omega \neq 0$. Now we generalize our dc results to finite frequency ω to consider dynamics of resistances. The screened conductance g_ω^s in Eq. (6) becomes complex and depends not only on τ_d [34], but also on $\tau_{RC} = C_\mu/\mathrm{tr}\,\Gamma$. Here C_μ is the electro-chemical capacitance, which relates the charge of the dot to the chemical potentials, it is defined by the Coulomb interaction strength, $1/C_\mu \equiv 1/C + \Delta/\nu_s$ [25]. A frequency $\sim 1/\tau_{RC}$, when the capacitor and the contacts conduct similarly, is then a natural characteristic of a two-terminal measurement. The finite-frequency correlations and WL of conductance were considered only for ballistic QPCs at $T, \omega \to 0$. Quantum effects in ac transport depended only on $\omega\tau_d$ *at low frequencies* [21], but at higher ω they start to depend on $\omega\tau_{RC}$ as well. In contrast, for R_{2t} our Eq. (2) shows that *at any* frequency the WL is independent of τ_{RC}; this is compatible with τ_{RC}-dependent WL in *conductance* [21], because τ_{RC} is eliminated by the matrix inversion. The reason is that WL only increases the return probability, but does not redistribute charge.

Low-frequency conductance correlations can be generalized to arbitrary frequencies ω, ω' using parameter $z \equiv -i\omega\tau_{RC}/(1 - i\omega\tau_{RC})$ and z' introduced similarly, and considering ballistic QPCs with $n_i = N_i/N$ for simplicity. For $\beta = 2$ we find the correlations,

$$\frac{\langle g_{ij,\omega}^s g_{kl,\omega'}^s \rangle}{\nu_s^2 n_i n_j n_k n_l} = \int d\tau \frac{(2\pi^2 T)^2 e^{-\tau/\tau_d}(e^{i\omega\tau} - 1)(1 - e^{i\omega'\tau})}{\tau_d \omega\omega' \sinh^2(2\pi^2 T\tau)}$$
$$\times \left[\frac{(\delta_{ik}/n_k - 1 + zz')(\delta_{jl}/n_l - 1 + zz')}{1 - i\tau_d(\omega + \omega')/2} - \frac{\tau z^2 z'^2}{\tau_d^3 \omega\omega'} \right], (7)$$

and for $\beta = 1$ the same r.h.s. with $k \leftrightarrow l$ should be added. Results of Ref. [21] are reproduced by differentiation $\partial_{\omega(\omega')}$ of Eq. (7) at $\omega, \omega' = 0$. Using Eq. (7) we find that fluctuations of R_{2t} in a dot with ballistic probes, $N = N_1 + N_2 + N_f$, given by Eq. (4) manifestly depend on both τ_d and τ_{RC}. Only the four-probe scheme can show if $\omega\tau_{RC}$ is an intrinsic scale of the dot itself, or if it comes from the chosen two-terminal measurement scheme. In other words, does the frequency-scale $1/\tau_{RC}$ survive in resistance statistics if the role of contacts is minimized? Equation (3) for R_{4t} does not depend on τ_{RC}, and we conclude that this scale is extrinsic and can be understood as a contact property. Since Gaussian R_{4t} is fully characterized by Eqs. (1,3), we conclude that the quantum transport at *arbitrary* frequency ω is insensitive to screening only in the four-terminal scheme.

Experimental relevance. Recently, Lerescu *et al.* [18] measured statistics of R_{4t} in dc-biased ballistic dots and compared with numerical data for $N_f = 0$. The rms R_{4t} qualitatively agreed with the numerics, but was about 20

times smaller than they expected. These data at $N \gg 1$ indeed correspond to our predictions that $\langle R_{4t} \rangle = 0$ and its fluctuations are strongly diminished by an increased voltage probe coupling. Their numerical data are well-fitted by Eq. (3), which demonstrates a weak dependence on $N_3 - N_4$ and does not depend on $\beta = 1, 2$. Reasons for the much reduced experimental $\langle R_{4t}^2 \rangle$ are strong decoherence, which might depend on couplings, and possibly direct source-drain trajectories, which result in voltage drop fluctuations smaller than expected by RMT.

High-frequency experiments are more difficult. If a leakage current through C bypasses the drain, we have $I_{1\omega} \neq -I_{2\omega}$. Resistance R_{4t} relates the voltage drop V_{34} to some linear combination of these currents, and our R_{4t} corresponds to $(I_{1\omega} - I_{2\omega})/2$. If the voltage drop is measured via a transmission line with length comparable to $2\pi c/\omega$, the circuit should be also taken into account. The phase shift of transmitted voltage with respect to current source has a classical component due to the circuit and a fluctuating part ϕ due the dot, and we consider its mesoscopic distribution $P(\phi)$. If the line impedance between the exits 3,4 (1,2) is large compared to $R_{4t}(R_{2t})$, one has $\phi = \arg R_{4t}$. Using Eqs. (3,5) and $\langle |R_{4t}|^2 \rangle$ calculated similarly, and normalizing $\oint d\phi P(\phi) = 2\pi$, we find

$$P(\phi) = \frac{\sqrt{\int_{\omega 0}^2 - |\int_{\omega\omega}|^2}}{\int_{\omega 0} - \text{Re}\left[\exp(-2i\phi)\int_{\omega\omega}\right]} = \frac{\sqrt{a(a+1)}}{a + \sin^2(\phi - \phi_0)} \quad (8)$$

This π-periodic distribution is insensitive to the probe properties and dephasing. The position ϕ_0 and the height of the distribution maximum are defined by Eq. (8), and Fig. 1(b) presents $P(\phi)$ for low $T \ll \hbar/\tau_d$. The maximum $\sqrt{2}/\omega\tau_d$ at $\phi_0 = \omega\tau_d$ for $\omega\tau_d \ll 1$ is sharp and the reactive part of R_{4t} is small. At $\omega\tau_d \gg 1$ the peak at $\phi_0 = \pi/2 - \log_2 \omega\tau_d/(4\omega\tau_d)$ is small due to weak correlations between (re)active parts, and any phase ϕ is possible. For high $T \gg \hbar\omega, \hbar/\tau_d$ when RMT is still valid, one has $P(\phi) = \omega\tau_d/[\sqrt{1 + \omega^2\tau_d^2} - \cos(2\phi - \arctan \omega\tau_d)]$, qualitatively similar to the low-T result. We expect that the universal distribution given by Eq. (8) can be found from phase measurements by varying either ω or T.

Conclusions. We discuss statistics of multi-terminal transport through a well-conducting chaotic quantum dot with imperfect coupling. Using RMT we find that four-probe resistance is unaffected by weak localization and its fluctuations are governed only by the dwell-time. Unlike two-probe transport, the four-terminal one is insensitive to the Coulomb interactions at any frequency, suggesting that the charge-relaxation time is connected to the measurement scheme. The fluctuations are given by a single analytical expression for arbitrary temperature, ac frequency, non-ideal coupling of the contacts, and floating probes. We propose a universal mesoscopic distribution for the phase of voltage transmitted through the dot.

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